

Be sure to clearly write all of your answers in complete sentences.

1. State the definitions of the following types of rings.
 - a. Ring
 - b. Ring with identity
 - c. Commutative Ring
 - d. Integral Domain
 - e. Field
 - f. Subring
 - g. Ideal

2. Determine what type of ring(s) the following sets are. Note: a set may be more than one type of ring. If so, list every type of ring that is it. A set may also not be a ring at all. If so, state which ring axiom fails.
 - a. $2\mathbb{Z} = \{2n \mid n \in \mathbb{Z}\}$
 - b. \mathbb{Z}_{19}
 - c. $\mathbb{Z}_{12}[x]$
 - d. $M_{2 \times 2}(\mathbb{Z}_2)$, the set of all 2×2 matrices entries in \mathbb{Z}_2
 - e. $\left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \mid a, b, c \in \mathbb{Z} \right\}$
 - f. Define a new addition and multiplication on \mathbb{Q} by $r \oplus s = r + s + 1$ and $r \odot s = rs + r + s$
 - g. $\mathbb{Z}_5[x]/\langle x^2 + 1 \rangle$
 - h. $\mathbb{Z}_3 \times \mathbb{Z}_3$

3. State the definitions of the following types of elements. Give an example of each type of element. Be sure to clearly state the ring in which your element lives.
 - a. Zero divisor
 - b. Unit
 - c. Additive identity
 - d. Multiplicative identity

4. State the definitions of the following types of functions. Give an example of each type of function. Be sure to clearly state the rule, the domain, and the image of the function (ie: what are the rings R and S of $f: R \rightarrow S$).
 - a. Injective function
 - b. Surjective function
 - c. Bijective function
 - d. Homomorphism
 - e. Isomorphism